Institute of Mathematics
College of Science
University of the Philippines Diliman

Mathematics 197 (Special Topics)
Selected Topics in Complex Analysis

Course description Infinite sequences and series of complex-valued functions, infinite products, infinite product representation of functions, entire functions, conformal mappings and their properties, linear fractional transformations, Schwarz lemma, Blaschke products.

Pre-requisite and Credit Math 128 or equivalent; 3.0 units

List of Topics

I. Review of basic complex analysis
   A. The complex numbers
   B. Holomorphic functions and their properties
   C. Sequences and series of complex numbers
II. Infinite sequences and series of functions
   A. Pointwise and uniform convergence
   B. Weierstrass M-test
III. Infinite products
   A. Infinite products of constant factors
   B. Infinite products of functions
   C. Logarithmic differentiation
   D. Infinite product representation for the sine function
IV. Special Functions
   A. Gamma function
   B. Stirling's approximation formula
   C. Riemann zeta function
   D. Euler product theorem and the infinitude of primes
V. Entire Functions
   A. Elementary factors
   B. Weierstrass factorization theorem
VI. Conformal Mappings
   A. Basic properties of conformal mappings, Riemann mapping theorem
   B. Linear fractional transformations
   C. Composition of elementary maps
   D. Schwarz's lemma
   E. Automorphisms of the unit disc
   F. Blaschke products
   G. Factorization of bounded analytic functions on the unit disc

References

1. N. Asmar and K. Grafakos, *Complex Analysis with Applications*, Springer (2018).
2. J. Conway, *Functions of One Complex Variable I (2nd Edition)*, Springer (1978).
3. T. Gamelin, *Complex Analysis*, Springer (2001).
4. M. Gonzalez, *Classical Complex Analysis*, Marcel Dekker Inc. (1992).
5. M. Gonzalez, *Complex Analysis – Selected Topics*, Marcel Dekker Inc. (1992).
6. D. Marshall, *Complex Analysis*, Cambridge University Press (2019).
7. R. Michel, *The (n+1)th proof of Stirling's formula*, The American Mathematical Monthly, 115:9 (2008), 844-845.
8. S. Ponnusamy and H. Silverman, *Complex Variables with Applications*, Birkhäuser (2006).
9. E. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press (2003).
10. J. Taylor, *Complex Variables*, American Mathematical Society (2011).