Computations with Modular Forms
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1. **Hecke operators and eigenforms.**

   (a) Use the `mfheckemat` command to compute Hecke operators T_n in S_{24}. Verify that $T_n T_m = T_m T_n$ for a few pairs of coprime integers m and n.

   (b) Use the `mfeigenbasis` and `mfcoefs` commands to compute the q-expansions of the normalized eigenforms, say f_1 and f_2, in S_{24}.

   (c) Verify computationally that the eigenvalues of T_n are the nth Fourier coefficients of the eigenforms f_1 and f_2.

   (d) Compute the characteristic polynomial of the Hecke operator T_2 of S_k for several values of k. Make a conjecture. What would this imply for the coefficients of the (normalized) eigenforms in S_k?

   (e) Find the q-expansions of the (normalized) eigenforms in $S_2(\Gamma_0(26))$.

2. **Values of the modular j-function.** Let $K = \mathbb{Q}(\sqrt{D})$ where $D < 0$ and squarefree. Let

$$w_D = \begin{cases} \sqrt{D} & \text{if } d \equiv 2, 3 \pmod 4 \\ (1 + \sqrt{D})/2 & \text{if } d \equiv 1 \pmod 4 \end{cases}$$

   so that $\mathcal{O}_K = \mathbb{Z}[w_D]$.

   (a) Compute $j(w_D)$ for several values of D.

   (b) Compute the class number of $\mathcal{O}_K$ for the same values of D.

   (c) Make a conjecture relating $j(w_D)$ and $\mathcal{O}_K$. Then test this conjecture for large values of $|D|$. (Example: Test your conjecture for $D = -163, -187, -211$.)