Computations with Modular Forms
11 January 2023

---

1. **Relations between Eisenstein series.**

   (a) Compute the q-expansions of E_4E_8, E_6^2 and E_{12}.

   (b) Find a linear relation between E_4E_8, E_6^2 and E_{12}.

   (c) (Optional) Translate this modular relation to an identity involving sum-of-powers-of-divisors functions.

   (d) Find other relations between Eisenstein series of different weights.

2. **Properties of $\tau(n)$.**

   (a) Compute $\tau(n)$, $\tau(m)$ and $\tau(mn)$ for several values of m and n. Make a conjecture.

   (b) Compute $\tau(p)$ for several primes p. Guess a function $f(p)$ such that $|\tau(p)| \leq f(p)$. Make it as sharp as you can.

   (c) Compute the dimension of $S_2(\Gamma_0(19))$.

   (d) Let $f(z) = \sum a(n)q^n \in S_2(\Gamma_0(19))$ such that $a(1) = 1$. Try 2(a) and 2(b), with $\tau(n)$ replaced by $a(n)$. Make conjectures.