

Course Title: Introduction to Ergodic Theory of Numbers

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The basic notions and theorems in Ergodic Theory will be introduced. Central are the concepts of measure preserving transformations, ergodicity, mixing, the pointwise ergodic theorem, various characterizations of ergodicity and the notion of isomorphism. Some popular numeration systems, such as decimal expansions, β -expansions, Lüroth series and continued fractions, will be viewed as ergodic systems and some important properties of these expansions will be investigated. We consider expansions in non-integer base. In this case almost every number (in the sense of Lebesgue measure) has uncountably many expansions. We introduce a transformation that generates all such expansions and exhibit its ergodic properties and as a result the properties of such expansions. We also zoom into the set of exceptional points having a unique, finite or countably many expansions in a given non-integer base. For such sets, we investigate their Hausdorff dimension.